

Supply chain coordination for a deteriorating product under stock-dependent consumption rate and unreliable production process

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Abstract

This article develops a supply chain coordination model with a single-vendor and a single-buyer. The vendor manufactures the product in lots and delivers to the buyer in equal shipments. However, the vendor's production process is not perfectly reliable. During a production run, the process may shift from an in-control state to an out-of-control state at any random time and produces some defective items. The buyer whose demand is assumed to be linear function of the on-hand inventory performs a screening process immediately after each replenishment. Moreover, the buyer's inventory is deteriorated at a constant rate over time. The vendor-buyer coordination policy is determined by minimizing the average cost of the supply chain. It is observed from the numerical study that channel coordination earns significant cost savings over the non-coordinated policy.

Keywords: Supply chain management, machine shift, stock dependent demand, and deterioration.

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1. Introduction

The strategic coordination between vendor (supplier) and buyer (retailer) has become one of the key issues in today's supply chain management. It has been recognized and verified in practice that supply chain performance cannot produce satisfactory result without smooth cooperation and collaboration among its members. When a vendor and a buyer are in a mutually agreed contract for a fixed time period, the problem of the buyer is how much quantity to purchase in each order. On the other hand, the vendor has to decide the economic production lot size and the optimal number of shipments to deliver the quantities to the buyer. One of the first works dealing with integrated vendor-buyer model is due to Goyal (1976) who developed a simple supply chain model of single vendor and single customer and their co-ordination. Banerjee (1986) developed a joint economic lot size model where the vendor produces the buyer's shipment size as a separate batch on a lot-for-lot basis assuming that the vendor's production rate is finite. Goyal (1988) relaxed Banerjee's lot-for lot assumption and proposed a more general economic lot size model to obtain a lower or equal joint total cost. Lu (1995) derived an optimal solution for the single-vendor single-buyer integrated model with equal shipments to the buyer. Goyal (1995) showed that different shipment size policy could give a better solution. His proposed policy involves successive shipments within a production batch increased by a constant factor which is equal to the ratio of production rate over the demand rate. Hill (1999) derived a structure of the globally optimal batching and shipping policy for the single-vendor single-buyer integrated production inventory problem. Using an interval search approach, Houge and Goyal (2000) developed a solution procedure for optimal production quantity in a single-vendor single-buyer production-inventory system with unequal and equal sized shipments from the vendor to the buyer under capacity constraint of the transport equipment. Ben-Daya and Hariga (2004) showed that coordination is effective from vendor's as well as buyer's perspectives for stochastic demand and variable lead time. Huang (2004) developed a model to determine an optimal integrated vendor-buyer policy in JIT (Just-In-Time) environment with unreliability condition. Wee and Chung (2006) proposed a two-echelon distribution-free integrated production-inventory model with imperfect process. They integrated the marketing and manufacturing channels assuming JIT deliveries. Li and Liu

(2006) used quantity discount policy to achieve coordination in a two-echelon supply chain where the demand met by the retailer is probabilistic. Kim et al. (2006) determined production allocation and ordering policy in a supply chain consisting of multiple plants and single retailer. Qin et al. (2007) considered volume discounts and franchise fees as coordination mechanism in a system consisting of a supplier and a buyer with price-sensitive demand. Zhou et al. (2008) addressed a two-echelon supply chain coordination model with one manufacturer and one retailer where the demand for the product at the retailer is dependent on the on-hand inventory. They developed the model when the manufacture follows a lot-for-lot policy. Sajadieh et al. (2009) contributed by developing an integrated vendor-buyer model in which the vendor delivers the production batch to buyer in n equal-sized shipments and the lead time between the vendor and buyer is stochastic. Yang (2010) provided the present value analysis for a similar type of vendor-buyer problem where the lead time is variable and the lead time crashing cost is a polynomial function of the length of lead time.

However, the above mentioned works didn't consider the effect of deterioration in retailer's inventory. Deterioration of goods like volatile liquids, fresh vegetables and fruits, radioactive substances, drugs, blood, etc. in the form of direct spoilage or damage, gradual physical decay in course of time, or obsolescence is a natural phenomenon and it has significant impact on the retailer's inventory policy. Ghare and Shrader (1963) first incorporated the possibility of deterioration in inventory modeling. They extended the classical EOQ model to consider a constant deterioration rate over time (exponentially distributed deterioration). A variable rate of deterioration has also been considered by Covert and Philip (1973) (Weibull distribution deterioration), Tadikamalla (1978) (gamma distribution deterioration), Moon and Lee (2000) (normal distribution deterioration), among others. There is a vast literature on deteriorating or perishable inventory. The readers can be referred to the review articles contributed by Nahmias (1982) and Goyal and Giri (2001) for details.

In this paper, we consider a two-echelon supply chain consisting of a single-vendor and a single-buyer. The vendor delivers the production lot to the buyer in n shipments. However, the vendor's production process is not perfect. During a production run, it may

shift from in-control state to out-of control state at any random time when it produces some defective items. The demand rate at the buyer is assumed as a linear function of the on-hand inventory. The buyer performs a screening process immediately after each replenishment. The objective of the study is to determine the optimal number of shipments from the vendor to the buyer and the optimal order size of the retailer in each replenishment so that the total cost of the supply chain is minimized. The paper is organized as follows. In the next section, assumptions and notations are given. In Section 3, mathematical models are developed from buyer's and vendor's view points and then the supply chain model is constructed. For a numerical example, the proposed supply chain model is demonstrated and the sensitivity analysis is carried out in Section 4. Finally, the paper is concluded in Section 5.

2. Notations and Assumptions

The following notations are used throughout the paper.

I : buyer's stock on display.

d : demand rate at the buyer; $d = a + bI$, where $a (\geq 0)$ and $b (> 0)$ are real constants.

p : vendor's production rate; $p > d$.

n : number of shipments from vendor to buyer in each cycle; n being a positive integer.

τ : time interval between successive replenishment at the buyer.

T : cycle time; $T = n\tau$.

Q : buyer's order quantity for each replenishment.

s_v : vendor's set up cost per set up.

s_b : buyer's ordering cost per order.

h_v : vendor's inventory holding cost per unit of inventory hold.

h_b : buyer's inventory holding cost per unit of inventory hold.

c : cost of unit item.

c_f : transportation cost per shipment.

c_s : unit screening cost.

c_w : warranty cost for each defective item.

x : screening rate.

θ : deterioration rate, $0 \leq \theta < 1$.

α : defective item production rate, $0 \leq \alpha < 1$.

t : random variable denoting the time to process shift.

t_m : production runs time of the manufacturer.

The assumptions made in this paper are as follows:

- (i) The supply chain consists of a single-vendor and single-buyer for a single product.
- (ii) Buyer's demand rate is a linear function of the on-hand inventory.
- (iii) Vendor's production rate is finite and uniform and is greater than the buyer's demand rate.
- (iv) During a production run, vendor's production process may shift from an in-control state to an out-of-control state at any time due to the occurrence of an assignable cause.
- (v) A fraction (α) of items produced after shift is non-conforming or defective. For each item, a warranty cost (c_w) is incurred to the vendor.
- (vi) Vendor's each production batch is transported to buyer in n shipments.
- (vii) Buyer performs a screening process immediately after each replenishment. The screening rate is so high that the number of perfect items is enough to satisfy the demand during screening time.
- (viii) Buyer's inventory deteriorates at a constant rate θ ($0 \leq \theta < 1$). Deteriorated units are neither repaired nor replaced.
- (ix) Shortages are not allowed in buyer's inventory.

3. Model Development

Let us suppose that the time to process shift at the vendor follows an probability distribution with probability density function $f(x)$. If $E(N)$ denotes the expected number of defective items produced during a production run then we have

$$E(N) = \alpha \int_0^{nQ/p} (nQ - pt)f(t)dt \quad (1)$$

<< Insert Figure 1 about here >>

3.1 Buyer's point of view

The variation of the inventory level at the buyer is depicted in Figure 1(a). If $I_i(t)$ ($i = 1,2$) denotes the inventory level at any time during the time interval $[0, \tau]$ then the instantaneous states of the inventory level are governed by following differential equations:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -a - bI_1(t), \quad 0 \leq t \leq t_1 \quad (2)$$

with $I_1(0) = Q$;

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -a - bI_2(t), \quad t_1 \leq t \leq \tau \quad (3)$$

with $I_2(\tau) = 0$.

Solving equations (2) and (3), we get

$$I_1(t) = Q \exp(-(\theta + b)t) + \frac{a}{b + \theta} [\exp(-(\theta + b)t) - 1], \quad 0 \leq t \leq t_1 \quad (4)$$

$$I_2(t) = \frac{a}{b + \theta} [\exp(b + \theta)(\tau - t) - 1], \quad t_1 \leq t \leq \tau \quad (5)$$

respectively. Since the screening process is completed at a time t_1 , the defective items are expected to be removed from the inventory at that moment. We, therefore, can write $I_2(t_1) = I_1(t_1) - E(N)$ which gives on simplification

$$\tau = \frac{Q}{x} + \frac{1}{b+\theta} \log \left(1 + \frac{(b+\theta)R(n,Q)}{a} \right) \quad (6)$$

where $R(n,Q) = Q \exp(-(\theta+b)t_1) + \frac{a}{b+\theta} [\exp(-(\theta+b)t_1) - 1] - E(N)$

Buyer's holding cost is given by

$$h_b \left[\int_0^{t_1} I_1(t) dt + \int_{t_1}^{\tau} I_2(t) dt \right] = \frac{h_b}{b+\theta} \left[\left(Q + \frac{a}{b+\theta} \right) \{1 - \exp(-(\theta+b)Q/x)\} \right. \\ \left. + \left(R(n,Q) + \frac{a}{b+\theta} \right) \{1 - \exp(\theta+b)(Q/x - \tau)\} - a\tau \right] \quad (7)$$

and the expected deterioration cost is given by

$$c \left[Q - E(N) - \int_0^{\tau} (a + bI_i(t)) dt \right] = c \left[Q - E(N) - a\tau - \frac{b}{b+\theta} \left\{ \left(Q + \frac{a}{\theta+b} \right) \left(1 - \exp\left(-(\theta+b)\frac{Q}{x}\right) \right) \right. \right. \\ \left. \left. + \left(R(n,Q) + \frac{a}{b+\theta} \right) \left(1 - \exp(\theta+b)(Q/x - \tau) \right) - a\tau \right\} \right] \quad (8)$$

Therefore, the buyer's average total cost which consists of ordering cost for n orders, screening cost, transportation cost, holding cost and deterioration cost is given by

$$ATC_b(n,Q) = \frac{s_b}{n\tau} + \frac{c_s Q + c_f}{\tau} + \frac{h_b - bc}{(b+\theta)\tau} \left[\left(Q + \frac{a}{b+\theta} \right) \{1 - \exp(-(\theta+b)Q/x)\} \right] \\ + \frac{h_b + c}{(b+\theta)\tau} \left[\left(R(n,Q) + \frac{a}{b+\theta} \right) \{1 - \exp(\theta+b)(Q/x - \tau)\} - a\tau \right] + \frac{C}{\tau} [Q - a\tau] \quad (9)$$

3.2. Vendor's point of view

The variation of the inventory level at the vendor is shown in Figure 1(b). The average holding cost of the vendor can be calculated as (Huang, 2004)

Average holding cost

$$\begin{aligned}
&= \frac{h_v}{n\tau} \left[\left\{ nQ(Q/p + (n-1)\tau) - \frac{nQ(nQ/p)}{2} \right\} - T\{Q + 2Q + \dots + (n-1)Q\} \right] \\
&= \frac{h_v}{2} \left[(2-n)Q^2 / (p\tau) + (n-1)Q \right]
\end{aligned}$$

Expected warranty cost per unit time is

$$\frac{c_w E(N)}{n\tau}$$

Therefore, the vendor's average total cost is given by

$$ATC_v(n, Q) = \frac{s_v}{n\tau} + \frac{h_v}{2} \left[(2-n)Q^2 / (p\tau) + (n-1)Q \right] + \frac{c_w E(N)}{n\tau} \quad (10)$$

3.3. Supply chain model

From equations (9) and (10), the integrated average total cost for the supply chain is given by

$$ATC(n, Q) = ATC_b(n, Q) + ATC_v(n, Q) \quad (11)$$

Our objective is to determine the optimal number of shipments n^* and the optimal order quantity Q^* for each shipment so that the average total cost $ATC(n, Q)$ is minimized. n being a discrete variable, for any given n , the necessary condition for optimum of $ATC(n, Q)$ is $\frac{d}{dQ} \{ATC(n, Q)\} = 0$ which gives for exponential distribution

$$\begin{aligned}
&\frac{h_b - bc}{(b + \theta)\tau} \left[\left\{ 1 - \exp\left(-\left(\frac{b + \theta}{x}\right)Q\right) \right\} + \frac{b + \theta}{x} \left(Q + \frac{a}{b + \theta}\right) \exp\left(-\frac{(b + \theta)Q}{x}\right) \right] + \frac{c_s}{\tau} + \frac{h_b + c}{(b + \theta)\tau} \\
&\times \left[\frac{dR}{dQ} \left\{ 1 - \exp(\theta + b)\left(\frac{Q}{x} - \tau\right) \right\} - \left(R + \frac{a}{\theta + b}\right)(\theta + b) \exp(\theta + b)\left(\frac{Q}{x} - \tau\right) - a \frac{d\tau}{dQ} \right] + \frac{c}{\tau} \left[(1 - n\alpha) \right. \\
&\left. - an \exp\left(-\frac{n\lambda Q}{p}\right) - a \frac{d\tau}{dQ} \right] + \left[\frac{s_b}{n} + c_s Q + c_f + c \left\{ (1 - n\alpha)Q + \frac{\alpha p}{\lambda} \left\{ \exp\left(-\frac{n\lambda Q}{p}\right) - 1 \right\} - a\lambda \right\} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{h_b + c}{b + \theta} \left\{ \left(R + \frac{a}{\theta + b} \right) \left(1 - \exp(\theta + b) \left(\frac{Q}{x} - \tau \right) \right) - a\tau \right\} + \frac{h_b - bc}{b + \theta} \left\{ \left(Q + \frac{a}{b + \theta} \right) \left[1 - \exp\left(-\frac{b + \theta}{x} \right) Q \right] \right\} \\
& \times \left(-\frac{1}{\tau^2} \right) \frac{d\tau}{dQ} + \frac{1}{\tau} \left[\frac{c_w}{n} \frac{dE(N)}{dQ} + \frac{2Q}{p} \left(1 - \frac{n}{2} \right) h_v \right] + \frac{n-1}{2} h_v + \left[\frac{s_v}{n} + \frac{c_w E(N)}{n} + \frac{Q^2}{p} \left(1 - \frac{n}{2} \right) h_v \right] \\
& \times \left(-\frac{1}{\tau^2} \right) \frac{d\tau}{dQ} = 0 \tag{12}
\end{aligned}$$

Where

$$\begin{aligned}
\frac{dE(N)}{dQ} &= \alpha n [1 - \exp(-\lambda n Q / p)] \\
\frac{d\tau}{dQ} &= \frac{1}{x} + \frac{1}{a + (b + \theta)R} \frac{dR}{dQ} \\
\frac{dR}{dQ} &= \left[1 - \frac{(b + \theta)Q}{x} - \frac{a}{x} \right] \exp(-(b + \theta)Q/x) - \frac{dE(N)}{dQ}
\end{aligned}$$

For a known value of n , equation (12) can be solved numerically by any one dimensional search method.

4. Numerical Example

To demonstrate the proposed model numerically, we consider the values of the parameters involved in the model as follows:

$a = 500$, $b = 0.02$, $p = 800$, $s_v = 150$, $s_b = 20$, $h_v = 0.2$, $h_b = 0.4$, $\theta = 0.02$, $c = 10$,
 $c_f = 25$, $c_s = 0.5$, $\alpha = 0.05$, $c_w = 5$, $x = 1000$ in appropriate units.

Example 1. Exponential process shift distribution $f(t) = \{\lambda \exp(-\lambda t), 0 \leq t \leq t_m$

Example 2. Uniform process shift distribution $f(t) = \left\{ \frac{1}{t_m}, 0 \leq t \leq t_m \right.$

Example 3. Normal process shift distribution $f(t) = \left\{ \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}, 0 \leq t \leq t_m \right.$

Assuming $\lambda = 100$ in Example1, the graph of the cost function $ATC(n, Q)$ for any given n and a wide range of values of Q is found to be convex. One instance is shown in Figure 2.

<< **Insert Figure 2 about here** >>

The computational results obtained in the line search technique are shown in the following table:

Table 1. Optimal results of the supply chain model for different values of n

n	Q^*	$ATC(n, Q^*)$
1	515.062	792.238
2	379.459	737.058
3	324.354	735.45
4	295.765	751.958
5	280.491	778.33
6	273.889	811.796

From the above table we see that optimal number of shipments is 3, the optimal order quantity of the buyer is 324.35 units and the corresponding average total cost is 735.45 units. Using equation (6), we then get the optimal cycle length of the buyer and that of the coordinated supply chain as $\tau^* = 0.608$ unit and $T^* = 1.824$ units, respectively.

Similar result follows for the other two distributions which are given in Table 2 and

Table 3. Where
$$E(N) = \int_0^{\frac{nQ}{p}} \alpha p \left\{ \frac{nQ}{p} - t \right\} \frac{1}{t_m} dt = \frac{\alpha n Q}{2}$$

Table 2. Optimal results of the supply chain model for different values of n

n	Q^*	$ATC(n, Q^*)$
1	515.773	712.574
2	375.103	640.418
3	315.211	620.262
4	281.291	616.002
5	259.657	618.917
6	245.088	625.756
7	235.136	635.009

Assuming $\mu = 0.01, \sigma = 4$ for example 3 we get

$$E(N) = \int_0^{\frac{nQ}{p}} \alpha p \left\{ \frac{nQ}{p} - t \right\} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(t-\mu)^2}{2\sigma^2} \right\} dt = \alpha(nQ - p\mu)$$

Table 3. Optimal results of the supply chain model for different values of n

n	Q^*	$ATC(n, Q^*)$
1	515.061	792.237
2	379.458	737.057
3	324.354	735.452
4	295.764	751.957
5	280.491	778.330
6	273.796	811.796

We will discuss some managerial implications in case of Example1. Let us now consider the situation for $n = 1$ i.e. lot for lot situation. If we consider the model from buyer's perspective, then the buyer's optimum order quantity is $Q_b^* = Q^* = 275.55$ units and the average total cost is $ATC_b^* = 435.0414$ units. Substituting $Q^* = 275.55$ in equation (10) we get $ATC_v^* = 436.4919$ units. Then the average cost for the coordination becomes $ATC_v^* + ATC_b^* = 871.5338$ units. This shows a cost reduction of 136.0838 units (871.5338-735.45) in the proposed supply chain model. Similarly, if we think of the model from vendor's perspective then we have the optimal order quantity $Q_v^* = Q^* = 1010.43$ units and the average total cost $ATC_v^* = 286.05$ units. Substitution of $Q^* = 1010.43$ in equation (9) gives $ATC_b^* = 598.69$ units. The average total cost in this non-coordinated policy is 884.74 units. This again shows that the coordination policy of the proposed supply chain model provides a cost reduction of 149.29 units than that for the lot-for-lot policy of the corresponding model.

We now compare the results of the coordinated policy with the non-coordinated policy for different values of the parameters b, θ, λ and α . For the non-coordinated policy, we follow the same procedure as above except taking $n = 1$. It is easy to see from Table 2 that the coordinated cost is always lower than the non-coordinated cost for different values of

the parameters. But the behavior of the cost reduction in the coordinated policy is not the same in all cases. When θ changes from 0.01 to 0.03, the cost reduction in the coordinated policy increases an amount ranging from 5% to 15%. For the parameters λ and b , the cost reduction gradually decreases as the values of λ and b increase. On the other hand, as α increases from 0.03 to 0.07, the cost reduction first increases (attains a maximum of 17% when $\alpha=0.06$) and then gradually decreases.

Table 4. A comparison of the coordinated and non-coordinated policies for different values of model-parameters.

Parameter	Value	Coordinated policy			Non-coordinated policy		
		n^*	Q^*	$ATC(n^*, Q^*)$	n^*	Q^*	$ATC(n^*, Q^*)$
θ	0.010	2	406.72	715.09	1	302.25	832.01
	0.015	2	392.37	726.26	2	259.08	753.01
	0.020	3	324.35	735.45	1	275.55	871.53
	0.025	3	314.82	744.54	2	238.00	778.89
	0.030	3	306.09	753.37	2	229.22	791.19
λ	50	3	321.81	733.34	2	246.63	777.54
	75	3	323.50	734.75	2	247.45	765.24
	100	3	324.35	735.45	2	247.87	767.18
	125	3	324.86	735.87	2	248.12	767.01
	150	3	325.19	736.15	2	248.28	767.10
b	0.010	3	325.73	733.61	2	248.86	764.01
	0.015	3	325.04	734.53	2	246.36	765.09
	0.020	3	324.35	735.45	2	247.87	766.18
	0.025	3	323.66	736.36	2	247.37	767.26
	0.030	3	322.98	737.28	2	246.88	768.34
α	0.03	4	283.34	639.91	2	244.24	686.87
	0.04	3	320.31	686.53	2	246.09	725.61
	0.05	3	324.65	735.45	2	247.87	766.18
	0.06	2	381.52	779.36	1	276.53	904.90
	0.07	2	384.64	823.75	1	277.50	939.01

5. Conclusions

This paper has considered a single-vendor single-buyer supply chain model where the consumption rate at the retailer depends on the on-hand stock and the production process at the manufacturer is not perfectly reliable. The process shift may occur during a

production run. As a result, the machine produces some defective items which have significant impact on the coordinated policy, as shown in the numerical analysis. It is observed in the numerical study that the coordinated policy provides lower cost than the non-coordinated policy in all circumstances. Future study could consider analyzing the model for multi-item and/or multi-vendor/buyer situation.

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